Numerical computation of electromagnetic fields in axisymmetric laminated media with hysteresis

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This work deals with the computation of transient electromagnetic fields in magnetic media with hysteresis. The classical Preisach model for hysteresis is considered. We assume axisymmetry of the fields. The magnetic field on the boundary of the domain is given as a source term. For the numerical solution, a space discretization by nodal finite elements and a backward Euler time-discretization are used. To deal with the non-linearities, we propose an iterative algorithm based on the properties of maximal monotone operators. The numerical scheme is validated with experimental results. In particular, we compare the eddy current and hysteresis losses obtained from the numerical computations with experimental ones.

***Index Terms*—Eddy currents, nonlinear magnetics, magnetic hysteresis, magnetic losses.**

I. Introduction

It is widely known that the performance of electric machines is mainly defined by the power losses. These losses include iron losses that are due to the magnetic field variations in the ferromagnetic materials composing the core of the machine. The efficiency, the thermal behavior and the compactness are some of the design constraints which are strongly influenced by the losses. Consequently, it is very important to predict them accurately for an optimum design of the device [2]. The iron losses can be divided into three main components: classical, hysteresis and excess losses, which are related to the intrinsic nature of magnetic materials. In the literature there are numerous publications devoted to obtain analytical simplified expressions to approximate the different components of these losses. These expressions are only valid under certain assumptions that do not hold in many practical situations. Numerical simulation is an interesting alternative in order to overcome these limitations and thus, in the last years, we can find several works focusing on this approach (see, for instance [1], [3]). The first step towards this end is the numerical solution of the underlying electromagnetic problem, which is the aim of this work.

II. Transient eddy current model with hysteresis

Eddy currents problems are modeled by the well known quasi-static Maxwell's equations. In many applications the computational 3D domain has cylindrical symmetry and all fields are independent of the angular variable. In such a case, in order to reduce the dimension and thereby the computational effort, it is convenient to consider a cylindrical coordinate system (r, θ, z) as shown in Fig. 1 and to write the magnetic field and the magnetic induction as $\mathbf{H}(r, z, t) = H(r, z, t)\mathbf{e}_\theta$ and $\mathbf{B}(r, z, t) = B(r, z, t)\mathbf{e}_\theta$, respectively. In such a case, taking into account the symmetry assumption and Ohm's law, Maxwell's equations can be written in terms of H as follows:

$$\frac{\partial B}{\partial t} - \frac{\partial}{\partial r}\left(\frac{1}{\sigma r}\frac{\partial (rH)}{\partial r}\right) - \frac{\partial}{\partial z}\left(\frac{1}{\sigma}\frac{\partial H}{\partial z}\right) = 0,$$

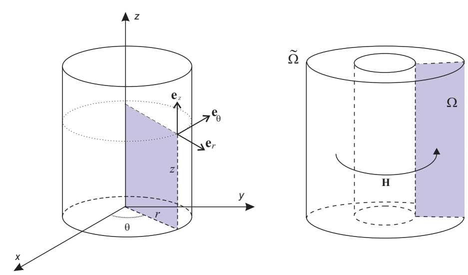

Fig. 1. Cylindrical coordinate system (left) and sketch of the domain (right).

where σ is the electrical conductivity of the material. This equation holds in a meridian section Ω of a 3D domain (see Fig. 1), for all time $t \in [0, T]$.

In ferromagnetic materials, where hysteresis phenomena may occur, the relation between B and H exhibits a history-dependent behavior and must be represented by a suitable scalar constitutive law accounting for hysteresis. We have chosen the well-known *classical Preisach model* (see [4]), in which the hysteresis operator is defined by

$$\mathcal{F}(H, \xi)(r, z, t) := \iint_{\alpha \leq \beta} \mathcal{R}_{\alpha\beta}(H(r, z, t), \xi(r, z))\, \mu(\alpha, \beta)\, d\alpha\, d\beta,$$

where μ is a distribution function with compact support that identifies the ferromagnetic material, ξ contains the information about the "initial state" of magnetization at each point (including eventually its history) and $\mathcal{R}_{\alpha\beta}$ is the relay function. Thus, we can write

$$B = \mathcal{F}(H, \xi).$$

To state a well-posed problem, we consider the following initial and boundary conditions

$$B(\cdot, 0) = B^0 \quad \text{in } \Omega, \qquad H = g \quad \text{on } \partial\Omega \times (0, T),$$

respectively, where B^0 and g are given data. For applications of this model, we refer for instance to [5].

By using nodal finite elements for space discretization and a backward Euler scheme for time discretization, we are led to the following discrete scheme:

Given B_h^0, find B_h^n and H_h^n, $n = 1, \ldots, m$, such that

$$\int_\Omega B_h^n G_h r \, drdz + \int_\Omega \frac{\Delta t}{\sigma^n r} \left[\frac{\partial(rH_h^n)}{\partial r} \frac{\partial(rG_h)}{\partial r} + \frac{\partial(rH_h^n)}{\partial z} \frac{\partial(rG_h)}{\partial z} \right] = \int_\Omega B_h^{n-1} G_h r \, drdz \quad \forall G_h : \; G_h = 0 \text{ on } \partial\Omega,$$

$$B_h^n(r,z) = \mathcal{B}^n(H_h^n)(r,z) \quad \text{in } \Omega,$$

$$H_h^n = g_h^n \quad \text{on } \partial\Omega,$$

where $\Delta t := T/m$, g_h^n is a convenient approximations of $g(t^n)$, and $\mathcal{B}^n$, $n = 1, \ldots, m$, is such that, given an initial state ξ,

$$\mathcal{B}^n(H_h^n)(r,z) := \mathcal{F}(H_h^{\Delta t}, \xi)(r,z,t^n), \quad (r,z) \in \Omega,$$

with $H_h^{\Delta t}$ being the piecewise linear time-interpolant such that

$$H_h^{\Delta t}(r,z,t^l) = H_h^l(r,z), \quad (r,z) \in \Omega, \quad l = 0, \ldots, n.$$

Different algorithms have been proposed to approximate the non-linear equation with hysteresis (see, for instance, [1]). Here, we propose an iterative fixed point type algorithm presented in [6], which is based on the properties of maximal monotone operators and their Yosida regularization. Thus, the previous problem can be reformulated as follows:

Given B_h^0, find H_h^n and q_h^n, $n = 1, \ldots, m$, such that

$$\int_\Omega \beta H_h^n G_h r \, drdz + \int_\Omega \frac{\Delta t}{\sigma^n r} \left[\frac{\partial(rH_h^n)}{\partial r} \frac{\partial(rG_h)}{\partial r} + \frac{\partial(rH_h^n)}{\partial z} \frac{\partial(rG_h)}{\partial z} \right] + \int_\Omega q_h^n G_h r \, drdz = \int_\Omega B_h^{n-1} G_h r \, drdz \quad \forall G_h : \; G_h = 0 \text{ on } \partial\Omega,$$

$$q_h^n = \mathcal{B}_\lambda^{n,\beta}(H_h^n + \lambda q_h^n) \quad \text{in } \Omega,$$

$$H_h^n = g_h^n \quad \text{on } \partial\Omega,$$

where $\mathcal{B}_\lambda^{n,\beta}(G)$ is the so-called Yosida regularization of $\mathcal{B}^{n,\beta}(G) := \mathcal{B}^n(G) - \beta G$ and λ and β are real numbers that have to be chosen such that $0 < \lambda\beta \leq 1/2$.

At each iteration of the fixed point algorithm, q_h^n has to be updated at particular integration points in Ω. This is done by solving a scalar non-linear equation for each point. Thus, at each iteration step, we have to solve a linear system and one scalar nonlinear equation per integration point. An interesting feature of the proposed algorithm is that, in cases where σ is time independent and the same at all iteration steps, the matrix associated to the linear problem is always the same and thus can be assembled (and eventually factorized) only once before the time step loop.

III. Numerical Example

To assess the validity of the proposed numerical approach we have solved the above problem with source terms obtained from physical measurements done on an Epstein frame considering a material sheet of thickness 0.5 mm, width 30 mm, and electrical conductivity 4064777 $(\text{Ohm/m})^{-1}$, subjected to a sinusoidal flux excitation with frequencies f and induction peak levels B_m specified in Table I. For each of these values, the physical measurements were the magnetic field on the boundary of the sheet and the total electromagnetic losses per cycle and per unit volume. To simulate the experimental setting with our axisymmetric model, we considered a rectangular domain $\Omega = [R_1, R_2] \times [0, d]$ with $R_1 = 100$, $R_2 = 100.03$ and $d = 0.0005$ (meters). Then, we numerically compute the total electromagnetic losses by summing up the eddy current ($\boldsymbol{L}_E$) and hysteresis losses ($\boldsymbol{L}_H$),

$$\boldsymbol{L}_E := \frac{1}{\pi d(R_2^2 - R_1^2)} \left\{ \int_0^T \left[2\pi \int_\Omega \frac{|\text{curl}(H_h \mathbf{e}_\theta)|^2}{\sigma} r \, drdz \right] dt \right\},$$

$$\boldsymbol{L}_H := \frac{1}{\pi d(R_2^2 - R_1^2)} \left\{ 2\pi \int_\Omega \left[\int_0^T \frac{\partial B_h}{\partial t} H_h \, dt \right] r \, drdz \right\},$$

and we compare them with the total measured losses. The results are summarized on Table I.

TABLE I
Total losses (J/m^3)

f(Hz)	B_m(T)	$\boldsymbol{L}_E$	$\boldsymbol{L}_H$	Total (exp.)	Relative error(%)
25	0.5	9.9144	126.2274	121.2594	12.2732
	0.9	32.7726	269.4965	300.2454	0.6740
	1.4	91.3716	585.7265	638.9281	5.9741
150	0.5	48.2460	146.0968	167.5503	15.9907
	0.9	178.5306	282.2810	459.1568	0.3603
	1.4	506.3943	588.1854	1090.178	0.4037

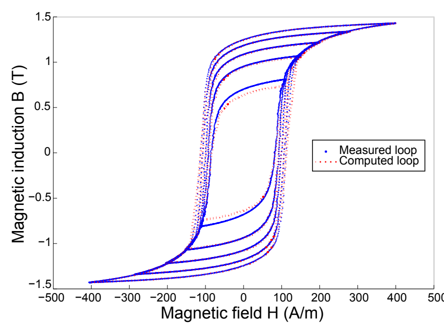

Fig. 2. Comparison between the numerically computed and the measured hysteresis loops.

IV. Conclusions

The results summarized on Table I show a good agreement between the numerical and experimental results. The largest discrepancy is observed for small values of the peak values, which is consistent with the fact that the inner hysteresis loops are worse approximated than the exterior ones (see Fig. 2).

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